“Compressed Sensing” for Rapid MR Imaging

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MRSRL People
MR Imaging

- Non-radiation Non-toxic imaging modality.
- Flexible tissue contrast
- Arbitrary plane imaging
- Many applications
- Still Too Slow
Outline

- A short “tour” of k-Space
- “Compressed Sensing”
- “CS” and MRI + applications
A Tour in k-Space
Signal Equation

Magnitude (Unknown)

Phase (known and controlled by linear gradient magnetic fields)
k-Space

k-space

\[ k_x \]

\[ k_y \]

image-space

Fourier
k-Space

Compressed Sensing for rapid MRI

k-space

image-space

Fourier
k-Space

k-space

image-space

k_x

k_y

Fourier

Compressed Sensing for rapid MRI
Applying linear magnetic field gradients controls the phase.

Like taking a “tour” in k-space
k-Space Trajectories

Cartesian:

- Used routinely in almost 99% of sequences.
- Extremely robust.
- Whenever possible, use this!
k-Space Trajectories

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• Extremely robust.
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Spirals:

• Used mainly for real-time and rapid imaging
• Fast & hardware efficient.
k-Space Trajectories

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- Extremely robust.
- Whenever possible, use this!

Spirals:
- Used mainly for real-time and rapid imaging
- Fast & hardware efficient.

Radial:
- Specific applications (Hi res, flow…)
- Almost always under-sampled
Pulse Sequence

Cartesian Trajectory

Spiral Trajectory
Hardware constraints & Physical Imperfections

• Hardware issues:
  – Gradients limited by maximum amplitude and slew rate.

• Physical issues:
  – Nerve stimulation
  – Eddy currents ⇒ Trajectory deviations
  – Trajectory dependent artifacts

\[ |k_t| < G_{\text{max}} \]
\[ |k_{tt}| < S_{\text{max}} \]
k-space Sampling

It’s all about finite support!

Fourier Transform
K-space Sampling

Finite support in frequency

Fourier Transform

Resolution
K-space Sampling

Finite support in frequency

Fourier Transform

Resolution

Compressed Sensing for rapid MRI
K-space Sampling

Finite support in frequency

Fourier Transform

Resolution
K-space Sampling

Finite support in Space

FOV

Fourier Transform

$\Delta k = 1/\text{FOV}$
K-space Sampling

Finite support in Space

FOV

Fourier Transform

$\Delta k = \frac{3}{FOV}$
Non Cartesian K-space Sampling

Non-Uniform Fourier Transform

Δk = 1/FOV

FOV
Non Cartesian K-space Sampling

Non-Uniform Fourier Transform

$\Delta k = \frac{3}{FOV}$
Here are the Important points:

- In MRI, sampled are collected in the Fourier domain (k-space).
- Sampling is Controllable (… but with restrictions)
- scan time $\propto$ # samples
“Compressed Sensing”
Sparsity

- But what if the image is sparse?

Sparse in Space

Sparse in time

Sparse in a transform domain

wavelets

Finite differences
Compressibility

Compressed Sensing for rapid MRI

Lossless or visually lossless compression
“Compressed Sensing”
A Surprising Experiment

* E.J. Candes, J. Romberg and T. Tao.
A Surprising Result*

* E.J. Candes, J. Romberg and T. Tao.
“Compressed Sensing”

First Presented by:


- D. Donoho “Compressed Sensing”
"Compressed Sensing"

Basic idea:

- **Sparse/Compressible** signals can be accurately recovered from highly incomplete *random Fourier* samples.

- Recovery by solving a *non-linear* convex optimization problem.
CS Formulation

\[
\text{minimize} \quad \| \Psi m \|_1 \\
\text{s.t.} \quad \| Fm - y \|_2 \leq \varepsilon
\]

- \( m \) – image
- \( F \) – Random Fourier Sampling Operator
- \( y \) – Measurements
- \( \Psi \) – Sparsifying transform
Reconstruction Formulation

\[
\text{minimize} \quad \|\Psi m\|_1 \\
\text{s.t.} \quad \|Fm - y\|_2 < \varepsilon
\]

\(m\) – image
\(F\) – Random Fourier Sampling Operator
\(y\) – Measurements
\(\Psi\) - Sparsifying transform

Randomly under-sampled Fourier Transform
Enforces Data Consistency
Reconstruction Formulation

\[
\text{minimize } \| \Psi m \|_1 \\
\text{s.t. } \| Fm - y \|_2 < \varepsilon
\]

\[\|x\|_1 = \sum x_i \] - Crucial for the reconstruction!

\[m \text{ – image} \]

\[\Psi \text{- Sparsifying transform (finite differences, wavelet)} \]
Why Does it Work?

….. And why random?

Let’s think simple,
- 1D signal.
- Sparse in signal domain.
Equispaced VS Random

Signal:

K-space:

- Equi-spaced sampling
- Random sampling

Compressed Sensing for rapid MRI
Equispaced VS Random

Signal:

K-space:

Equi-spaced sampling

random sampling

Inherent ambiguity
Equispaced VS Random

Signal:

K-space:

Equi-spaced sampling

random sampling

Inherent ambiguity

Buried in interference

“Random noise”
Equispaced Vs Random

Equispaced under-sampling

Random under-sampling

Inherent ambiguity

Burried in interference

“Random noise”
Sparse Recovery from Random Sampling

Threshold
Sparse Recovery from Random Sampling

- Threshold
- Get coefficients
Sparse Recovery from Random Sampling

- Threshold
- Get coefficients
- Compute interference
Sparse Recovery from Random Sampling

- Threshold
- Get coefficients
- Compute interference
- Compute residual
Sparse Recovery from Random Sampling

- Threshold
- Get coefficients
- Compute interference
- Compute residual
- Threshold......
Sparsity Revisited

Sparse in Space

Sparse in a transform domain

Finite differences

wavelets

Compressed Sensing for rapid MRI
Here are the Important points:

- For “Compressed Sensing” to work we need:
  - Sparsity
  - Incoherent interference between the transform coefficients (random partial Fourier).
  - Sparsity based reconstruction.
Compressed Sensing and MRI

Practical schemes for CS in Magnetic Resonance Imaging
CS and MRI

• CS
  – Signals are sparse/compressible.
  – Random partial Fourier sampling.
  – Small number of measurements.

• MRI
  – MR Images are often compressible.
  – Control on Fourier sampling.
  – Scan time proportional to # samples.
CS and MRI

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Practical Schemes for random sampling

“Randomness is too important to be left to chance” *

*Robert R. Coveyou, Oak Ridge National Laboratory
Practical Schemes for random sampling

- “Pure random” sampling is impractical in MRI.

- Instead, design “effectively random” sampling.
  - Incoherent aliasing interference
  - Efficient for hardware and application
  - Robust

- Tailor trajectory for application (Cartesian, spiral...)
- Randomly perturb to be “effectively random”.
Talk about.....

- Randomized 2D spirals
  - Whole heart imaging with a single breath-hold

- Randomized 3D Cartesian
  - Angiography
  - Teaser: Dynamic cardiac imaging
Perturbed Spirals

Spirals are:

- Fast and efficient.
- Irregular sampling.

Coherent interference
Perturbed Spirals

Spirals are:
- Fast and efficient.
- Irregular sampling.

Introduce randomness:
- Deviating from the analytic spiral – costs a little.
- Perturbing angle – for free
- Can also use Variable density – more irregular
Coherency

Randomness is GOOD!
Compressed Sensing for rapid MRI

Phantom Scans

Min norm  CS: Total Variation  CS: L₁ Wavelet

19/34 interleaves perturbed spiral
Nominal FOV 16cm
Resolution 1mm
3ms readout, GRE sequence
Whole Heart, Single Breath-hold Coronary Arteries MR Angiography*

- 2005: Heart related diseases #2 killer in the US. (#1 till 2004) and #1 in Israel.

- High-res cardiac imaging is very challenging
  - Unfortunately (.. Or not), the heart moves.
  - People insist on breathing!

- Many approaches, ...however,
  - Long scan time (10-30 min).
  - Partial coverage

* with Juan Santos
Whole Heart Single Breath-Hold Imaging

Start breath-hold

Short time frame

Real-time localization

Delay

Trigger

Slices

Slice 1  Slice 2  …  Slice N
Coronary Imaging

single breath-hold whole heart at 3T
17 Itlv Variable density spirals ~50%
5.6ms readout
Nominal FOV 20
0.8mm resolution

Talk about.....

- Randomized 2D spirals
  - Whole heart imaging with a single breath-hold

- Randomized 3D Cartesian
  - Angiography
  - Teaser: Dynamic cardiac imaging
3D randomized Cartesian
3D randomized Cartesian

- Randomly decimating phase-encodes is free.
- Purely random in 2D.
- Scan time in 3D is an issue.
- Robust trajectory.
- Easy to implement.
- Separability of kz-ky and kx.
- Many applications.
3D randomly under sampled Cartesian angiography

- Angiograms are truly sparse.
- Very bright blood vessels, low background… perfect!
- Blood vessels are piece-wise constant, use Total-Variation
- Speed essential, especially for contrast enhanced
Results Angiography

- Non-contrast angiograms
- T2 prep with fat saturation.
- Data under-sampled retrospectively to 33%.
- 128x128x384 matrix, each slice reconstructed separately
- Recon with Total Variation.

100%  33% CS
Results Angiography

- Non-contrast angiograms
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- Data under-sampled retrospectively to 33%.
- 128x128x384 matrix, each slice reconstructed separately
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More Angiography

Compressed Sensing for rapid MRI
More Angiography

Narrowing

Small vessels disappear

Compressed Sensing for rapid MRI
Teaser
Hi Frame-Rate Dynamic Imaging

- Imaging a dynamic scene, results in aliasing in spatial – temporal frequency space

- Very hard application
Hi Frame-Rate Dynamic Imaging

- But, Smooth & periodic signals are sparse in wavelet-fourier space

- Can we exploit sparsity?
Hi Frame-Rate Dynamic Imaging

- Cartesian 3D can be a 2D spatial 1D temporal.
- Randomize ordering! (For Free)
Hi Frame-Rate Dynamic Imaging

- Reconstruction at 4-fold acceleration (16x4ms / frame)
- Spatial transform: wavelets
- Temporal transform: Fourier
Hi Frame-Rate Dynamic Imaging

- TR = 4.5ms
- Res = 2.5mm
- Slice = 9mm
- 4.5 sec acquisition (1152*4.5ms)
- 7-fold acceleration (25FPS)
Main Points

- Find an impossible/important application.
- Make sure the signal is sparse.
- Design a robust & efficient randomized sampling scheme tailored to the application.
Conclusions

- Promising applications.
  - Exploit sparsity where sparsity exists.
- Choose application wisely.
- Transformations.
- Clinical trials – the Doc test.
The END

http://www.stanford.edu/~mlustig